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Metaphysics and Arithmetic

Introduction

This short essay is an adjunct to *Reading Sociologically* and should be read alongside it. In that essay, our objective was to introduce a heuristic *aide memoire* for reading sociological reports and then illustrate its use through some examples. In doing so, we pointed to various issues and challenges which writers of research reports face in managing the necessary transit across the abstraction gap between the phenomenology of the social data collected and the sociological phenomena research reports describe. Central to our discussion was the task of turning description of social objects gathered as data into descriptions of sociological objects cast in mathematical terms. At various points, we alluded to an allied difficulty which might also be present. This was the robustness of assuming an isomorphism between the logical grammars of our commonsense concepts of the social and that of the mathematics of the real number system $\mathbb{R}^N$ which is the basis of the arithmetic methods used in the analyses. Examining this isomorphism in the cases we were discussing would have taken us a long way off our intended path and further complicated what was already turning out to be a sufficiently complicated story.

This essay returns to that question. It looks beyond the immediate analysis of well posed problems to offer an initial exploration of the assumptions underlying the isomorphism. In doing so, it introduces a number of considerations which are relevant not just to mathematical sociology and the disciplinary metaphysics of the social sciences in general. These might be summarised in a blunt question: Why are we so sure the metaphysics of the social maps onto the structure of the Real Number system (or indeed either of its siblings, the Cardinal and Ordinal systems)?

Section 1. KF Structures

A common account of the difficulties in applying mathematics in Sociology rests on the potential for tension (or worse) between satisfying the mathematical requirements (for well-posed problems or whatever) and the sociological requirements for capturing the sociality of a phenomenon. But that simply describes the issue, it does not explicate it. To understand what it is about the mathematics and the sociology which generate the problem, we need to look at why applying mathematics works as well as it does in Physics[1] and why it so often doesn't in Sociology. Resorting to an obvious explanation by invoking a distinction between conceptual and empirical disciplines won't serve since both Physics and Sociology are empirical. We must look elsewhere and the most natural place to start is the conceptual structures (metaphysics) of the worlds organised by their descriptions and the logical grammars of the concepts deployed in them. Since the fit between the mathematics used and the phenomena it studies seems to be so tight in Physics, we build the 'base case' from that.

Our guide in this exercise is Penelope Maddy and her analyses of the metaphysics of Arithmetic, Logic and Physics. The approach Maddy adopts is what she calls "second philosophy".[2] It starts with what we ordinarily know about the world and, using the basic method of trial and error, works back through what various kinds of mathematics might have to say about that world. She partitions Mathematics into 'pure' or 'standard' and 'applied' forms. We will concentrate on her analysis of Arithmetic since it is at the base of most of forms of mathematics which Sociology has borrowed from the natural sciences. By 'applied' Maddy intends something like 'has a direct or untranslated application to the phenomena of the commonsense world'.

Maddy's narrative consists of commentary on the thinking of an idealised enquirer examining some philosophical topic or question. The idealisation, though, is post-Quinean. It is not the enquirer's ambition to argue from appearances to the apodicities underpinning grounded knowledge. Rather she starts from the assumption any well founded science tells us what there is in its domain and the task is to work out the grounds of such knowledge from what that science says

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[1] The phrase "as well" is open to an awful lot of debate which we will not enter into here. For our purposes, no matter how well or badly philosophers (and some physicists) think it works, it certainly appears to be a better fit for reasoning in Physics than it is for reasoning in Sociology.

[2] Maddy provides innumerable sketches of this approach [see Maddy 2000; 2007; 2011]. Whilst they are more or less the same, they do differ slightly in their details, a feature which has given her critics considerable ammunition. [Santos 2016]

and does. As we have said, her approach is one of probing, questioning and trial and error together with assaying the consistency and security of the answers she arrives at. As Maddy puts it:

> Very roughly, the thought is that the ground of a stretch of discourse is something extra-linguistic that guides and constrains what counts as proper or correct in that discourse, something extra-linguistic to which the discourse is responsive and responsible. [Maddy 2014, p. 223]

The second philosopher begins with the everyday world of material objects. A world in which there are shoes and ships, sealing wax, cabbages, kings and a whole lot more. She also starts with our science of that macro-world.[3] This world, she confirms, is one in which there are objects with properties. These objects stand in relations. This does not mean everything has the same properties or that all the distinctions between objects, properties and relations can be nailed down. She calls the underlying logic of these objects, properties and relations a 'KF-structure' because it articulates the conceptual integration of transcendental idealism, empirical investigation and formal logic laid down by Kant and Frege. The nub of her argument is a correspondence thesis. The metaphysics underpinning our common sense, scientific and mathematical understandings of the physical world are members of the set 'KF metaphysical structures'. This co-membership means scientific, mathematical and common-sense ways of understanding and describing the objects of the natural world correspond or stand in a mapping relationship to one another regarding the set defining properties.

Maddy's argumentational premise is that the world must have a KF-structure. As a result, our primitive cognitive mechanisms have evolved (or whatever developmental theory you want) to "detect and represent" objects which have these features [Maddy 2007 p 226]. It follows humans are cognitively configured (that is, think the way they do) because they live in such a world. Since science has its foundations in natural observation and engagement with the world, its descriptions evidence that world's KF-structure. Maddy is well aware this argument is an inference over non-philosophical evidence but that is how second philosophy operates. She is also well aware it would not satisfy the radical sceptic, but she dismisses radical scepticism on two grounds. First, she thinks scepticism is an exercise in nihilistic premise denial which, given her 'set-up conditions', she

[3] An important qualification. As we have already hinted, once you get beyond the macro-world, things get decidedly tricky and ugly. See Schapiro and Reeder [Shapiro 2009] for a commentary on Maddy and an examination of some of the implications of Quantum Physics for her argument.

cannot engage with. And anyway, *nothing* she could say would satisfy what scepticism is asking for.

With the KF structure of the world in place, Maddy turns to the other half of the practice underpinning science's success, the mathematical structure of Arithmetic. She finds its logic also shares a KF structure. Its operations look to be more or less complicated exercises using the number system to "track" (an important verb for Maddy) the manipulation of variously sized bundles of things. The trouble is the Real Number system ($\mathbb{R}^N$), for which Arithmetic is the operational mathematics, has one obviously problematic feature: infinite extensibility. There is no largest or smallest number. We are faced with what has been called the 'Benacerrafian challenge' [Benacerraff 1978]. How do we, a finite being operating with finite objects, ground the realism of the 'transfinite'? How do we justify the realism of adding objects together so they aggregate beyond the finite number of objects there are in our finite world? The general case of this recursion Maddy calls the '.....' problem. In a way, it is the nub of what Wittgenstein called the problem of "knowing how to go on".

Once again, Maddy doesn't offer philosophical arguments but psychological ones; the demonstration in Psychology of ontogenic mechanisms (or primitive number systems) for tracking small groups (less than 4) and large groups (more than 4). It is the first of these which allows us to 'track' individuals and is implicated in basic arithmetic. The second, called the "analog system", allows us to track larger groups but is somewhat imprecise. Studies have shown the same systems can be found among animals. Maddy takes this to mean there must be an evolutionary origin. But what humans have and animals lack is an elaborated language. Recursion is associated with one pertinent aspect of elaborated languages, the sequence of number words. Psychology has shown children learn recursion in learning number words. Here is Maddy's summary.

> .....it seems fair to say that by the time we master something like the decimal system, we have come to think that despite the limits of paper, pencil and human breath, there is always, at least in principle, another numeral..... Much as our primitive cognitive architecture, designed to detect KF-structure, produces our firm conviction in simple cases of rudimentary logic, our human language-learning device produces a comparably unwavering confidence in this potentially infinite pattern. [2014 p. 234].

We don't grasp infinity and then develop competence in the number system. We develop competence in the number system and eventually have what Jessica Carter [Carter 2019] calls an "epiphany" with regard to the possibility of infinity. It is this 'intuition' which gradually morphs into

the mathematicians' and philosophers' theorems about infinite sequence and is formalised in Peano's axioms for standard (infinitary as opposed to finitary) arithmetic.[4]

So here we have the story. Basic (finitary and applied) arithmetic and standard arithmetic are based upon (a) the common logic of KF-structures; and (b) the development of recursion as a linguistic competence. Standard (infinitary) arithmetic is the formal idealisation of the finitary form. Its idealisation is no more than abstract modelling which introduces necessary falsehoods, approximations and torsions. These can be justified only if they are benign and result in the provision of "mathematical depth" accessed and assessed in the usual ways that mathematicians do. In sum, for arithmetic

> ....firm groundings run throughout, from the world's KF-structures, to the recursive element of the language-learning devise, to the more esoteric facts of mathematical depth.[2014 p. 248]

As a form of mathematics, then, finitary applied arithmetic is secure. But what about the use science puts it to? Here Maddy does have some qualifications.

> When we represent a cannon ball as a perfect sphere, the lengths, times, angles and forces involved as real numbers, the local surface of the earth as flat, and so on, in order to determine where a given ball, fired with a given force, will land, we have a fairly good idea of at least some of our departures from literal truth and why they are admissible. When we represent spacetime as a continuous manifold, we aren't entirely sure whether or not this constitutes a literal truth, though our well-informed hunch is that even if it is an idealization, it's a good one—much as Euclidean geometry is a good approximation to the truth in most ordinary cases. But the fact remains that the mathematics has been peeled away from the science; the actual claim the scientist makes about the world is that it is probably, at least approximately, similar in structure to the mathematical model in certain respects, and that the idealizations involved are beneficial and benign for the purposes at hand.....
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> ...(O)ur best mathematical accounts of physical phenomena aren't the literal truths Newton took them for, but free-standing abstract models that resemble the world in ways that are complex and sometimes not fully understood. [2011, pp26-7]

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[4] In a discussion of structural theories of Mathematics, Brice Halimi [Halimi 2019], offers a fascinating example. Initial students of the theory of permutations usually are happy to accept that $\binom{abc}{bac}$ and $\binom{abc}{cba}$ are different permutations but the epiphany to see the same is true of $\binom{123}{213}$ and $\binom{123}{321}$ wholly eludes them. Clearly there is a lot we don't know about the psychology of numbers as representations.

The "peeling away" of the mathematics from the science opens a potential gap in the relationships. The mapping is now not correspondence but approximation. As a result, idealising across the gap must at least be non-distorting and should be structure preserving.

Section 2. The Elements of KF Structures

Any structure is an organisation of elements. KF-structures are no different. Their elements are objects, properties and relations, dependencies and their determinacy. We'll take a brief look at each in turn.

Objects

The material world consists of numerous (perhaps infinitely numerous) individually identifiable things. The paradigm for what is meant by 'thing' are the constituents of the macro-world in which we live. These are the medium sized physical objects around us such as trees, buses, jogging shoes, salt crystals and lumps of rock. These things have a physical unity, boundaries, positions in space and relative persistence over time. Each can be picked out from a collection of other things of the same type and from its surroundings. Most importantly, they can be counted and, if we have the appropriate methods, some of their properties can be scaled and measured. Many of their features fit the real number system and manipulations of those features by methods derived for the Real Number system reveal more.

As we move away from the paradigm ontological type, this characterisation begins to break down. In the domains of particle physics and the lower plants, objects do not necessarily comport themselves in the ways we expect. Particles such as photons don't seem to occupy a predictable, defined position and can appear to be smeared out in space. Under some circumstances slime moulds behave rather like plants and under others more like yeasts. The analysis of particles has led some to the conclusion that other forms of mathematics to those based in Arithmetic are needed for their description, whilst consideration of the lower plants and animals often results in ontological re-calibration and engineering. However, these kinds of adjustments do not lead anyone to propose the re-consideration of the paradigm cases.

Properties and Relations

One of the neighbours has a black and white dog which yaps incessantly. Another's has a morose look and is almost silent. Buses accommodate different numbers of passengers and are often painted different colours. Some properties are definitive of membership of a category. The black

and white dog is male because it has X and Y chromosomes. Sharing a single party wall defines a house as 'semi-detached'. Other properties while not definitive are intrinsic. Being rigid doesn't uniquely identify levers. Lots of other things are rigid too. But a rubber lever is not much use to anyone. Yet again, some properties are just contingent. It just so happens the blackbird's nest is in the rhododendron by the gate. It could have nested somewhere else.

Some of the properties an object has can affect other objects. Put your hand into a boiling saucepan and you will be scalded. Roll a marble down a slope and it will accelerate. These relationships can be direct as in the cases cited or indirect. Changing day length and increases in ambient temperature induce bio-chemical changes in frogs which in turn cause them to seek to spawn. Leaving the kitchen door open when baking bread causes the central heating thermostat in the hall to switch off as heat is transferred out of the kitchen. As a result, the bedrooms cool down. Interestingly, these indirect relationships are all mediated by local direct relationships (also known as 'mechanisms') thereby preserving the KF-structure of the relationships. As ever, move away from the domains of barking dogs, boiling saucepans and baking bread and the notions of distinctive properties and determinate relations become more and more unglued. Here we find talk of light waves/particles choosing their route through a slit screen or of causes happening after their effects have been manifest.

Dependence

As already indicated, one way in which objects are related is through the dependence of properties. For the kettle to boil, the switch on the wall has to be on. The carrots in the veg. patch won't survive unless rabbits from the neighbouring fields are kept out. Ivy looks a bit like her mother because she shares her mother's genetic makeup. Some of these relationships are binary. Left in the dark, green leaved plants die. Other dependences, though, are more like preferences. The foxgloves will reluctantly grow in the flower beds but spread like wildfire in the gravel pathways. Some of these dependences are visible and well understood. Eggs break (usually) when you drop them on the floor. Some are not, though if needed we have effective ways of figuring out what will happen (for example, the trial and error of tests and experiments). There again, others do escape understanding altogether. Why, for instance, does any loose length of string or any collection of clothes hangers get tangled up when you aren't looking at them?

INDETERMINACY

We have talked of objects being bounded and denumerable and for many, perhaps most, this is so. But for some it is not, or on occasion is not. Looking out of the window, the sky is grey but not uniformly so. Lighter and darker patches merge but where they merge is hard if not impossible to delineate. The field edges are defined by 'rough' grasses where the standard rye grass savanna has not been sown. Yet the boundary between the field edge and the 'improved' pasture is not a line but an imperceptible merging. While no-one has an official (or even unofficial) definition of the number of stones that make a way-marking cairn, when out on the hills we can usually tell the difference between a random pile of stones and a (constructed) cairn. 'More' and 'less', 'not enough' and 'sufficient' are essential concepts for us. But just how short does a piece of string have to be before it won't tangle?

Our ability to deal with indeterminacy is one of the commonsense ways we cope with the findings of the natural sciences. On one classic description, the desk under the keyboard is a solid, stable object. It was here yesterday and, as long as nothing untoward occurs, it will be here tomorrow much as it is now, covered in books, papers and other bits and pieces. At the same time, the table is just a lattice of forces and only has the appearance of solidity. Even the appearance of permanence is misleading. The desk is actually entropic. Given enough time, the physical structure will become disorganised. We can treat each of these descriptions as 'true' without worrying about at just what point we should switch from the adoption of one 'justified true belief' to another. Since, the home range of our concepts is common sense, we adopt the reasonable position of giving common sense usage a working level of primeordiality. This does not deny vagueness but embraces it. Vagueness is a sometimes feature of the world, a feature we don't have to give up on unless other ways of dealing with vague phenomena show benefit for managing our world. After all, does it help when you are looking for the pen you just put down to think of the table as a lattice of forces or imperceptibly sliding into a chaotic force field?

This is not a campaign for wholesale conversion to Maddy's proposed metaphysics. All we want to do is use her arguments to open up a novel line of discussion with regard to mathematical sociology. Maddy's argument turns on the confirmation by science that natural phenomena have KF structures of the kind common sense attributes to them. These structures are shared with Arithmetic. Our ordinary concepts about the natural world (and hence those of non-Quantum science) are KF constructions and so can be represented by appropriate methods of finitary arithmetic. Arithmetical procedures work in science because both the mathematics used and science share a common logic. Common sense understanding of the natural world also has that

logic. The explanations of mathematised science convince us because our two ways of thinking about the natural world are alike.

This conclusion leads to the obvious question. Can the same thing be said for thinking about the social world?

Section 3. KF Structures and the Social

What Maddy does is align the logical grammars of three distinct conceptual structures; common sense understanding of the world, pre-Quantum natural science (Physics) and basic arithmetic. Her claim is all three deploy KF-structures. This is an unsurprising but contingent consequence of the facts that:

1. Arithmetic and Physics are predicated in our common sense understanding of the world. They start with that but rapidly move beyond it.
2. Arithmetic and Physics evolved together as the applied mathematics of the natural world.

The 'ground' of all three forms of metaphysics is what some have called a 'mechanical' view of the world. The ultimate parts/units composing the world are 'unitary', 'solid', 'denumerable' 'objects' standing in 'causal' relations. All objects are either ultimate unitary objects of this kind (call them particles if you want) or made up from them. The KF-structure is the logic of these objects.[5] The question we are pursuing now is whether some or all our commonsense metaphysics of the social world *also* has a KF-structure. If it does, then importing of basic and standard arithmetic as part of the mathematical sociology may make perfect sense. If it doesn't, that move might amount to little more than a basic category mistake and lead to some of the difficulties we discuss in the companion essays to this. Of course, it could be some parts of the social do conform and some parts don't, with all the complications which would follow from that!

We came to our question as a result of looking at how well-posed investigative problems in Sociology might be. It is natural, then, to look at the process of constituting problems for investigation and analysis to see we can find any pointers to an answer. One location worth examining is what is known in methodology texts as "operationalisation" or what we called the use

[5] Whether there are 'looping' relationships between common sense understandings and the understandings of science as that science is popularised/bowdlerised is not germane for us right now. The basic grounding goes one way only.

of "analytical procedures": the translation of abstractions such as the sociological concepts of preference, healthy living and socially generous political cultures into systematically measurable indicator variables. From what we have said so far, it will be important to see if these indicator variables display reasonable adherence to K-F structures whilst maintaining their conceptual character as representations of the social objects for which they stand. If they do, mapping mathematical procedures onto them is structure preserving.[6]

This immediately brings us face to face with two important issues which we will have find ways to manage. The first is the fact our common sense and scientific categorisations of the natural world are of no importance to the objects we so categorise. Birds and bees do not care if we put them in the same or different ontological categories. As far as we know, not even our closest primate cousins, the Benobos, worry about what we think of their mental life or our assumption they act solely on the basis of biological 'drives'. The Ash trees in the garden are indifferent to our theories of how they measure time marked by the seasonal round. On the other hand, when Sociology talks about social categories and processes, sometimes what we say definitely does matter to those whose categories and processes they are.[7] The social worlds containing such phenomena are imbued with meaning for those who live in them. And these meanings count. What role such "meanings" should play in relating our sociological descriptions and their "understanding" of their ways of life is far from clear. Even after 150 years, we still don't know what to say about the sociological import of 'the actor's point of view', except that it seems to be important that it has one and it is important not to misrepresent it. At various times, we have talked about overcoming this challenge as "faithfulness to the phenomenon" where the phenomenon in question is the lived experience of those undertaking the courses of action under investigation. As we will see in Part II, questions of representation and misrepresentation of social actors' experience of the social are sometimes raised by the substitution of sociological for social characterisations. Closely related to these concerns are those relating to the inclusion of the observer/analyst within the analytical frame of reference. This is more than simply the reflexive extension to the practices of Sociology of ethnomethodological interest in practical reasoning. It involves the constitution of the

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[6] It is important to note that this is not a question of the logical validity of any imputed relationships between the indicator variables and the phenomenon. That is a very relevant question for the robustness of the explanations being given but it is not the question we are asking.

[7] The 'science wars' in and over SSST are more than sufficient evidence for this.

observer's objective analytic point of view in relation to the actor's subjective interpretive point of view. How do we frame the constructs of the analysis to carry both?

The second issue is how to treat what has recently become the highly popular sport of 'social ontology'.[8] Social ontology is a kind of sociological exercise. The home ranges of the categories of objects chosen for analysis are sociological and the accounts given of them (their 'grounding and 'anchoring' in Epstein's [Epstein 2015] case) are sociological.[9] This means what Epstein and others are up to should be labelled "sociological ontology" or "sociological metaphysics" both of which are perfectly proper things to be doing but they are not what we are after.[10] Mike Lynch [Lynch 2013] has called what we have in mind "ontography" , one justification for which was beautifully captured in a famous remark by John Austin.

> ...our common stock of words embodies all the distinctions men have thought worth drawing, and the connexions they have found worth making, in the lifetimes of many generations: these surely are likely to be more numerous, more sound, since they have stood up to the long test of the survival of the fittest, and more subtle, at least in all ordinary and reasonably practical matters, than any you or I are likely to think up in our arm-chairs of an afternoon—the most favoured alternative method. [Austin 1961] p.182]

What follows is nothing like a fully-fledged exercise in the ontography of the sociological mathematics of the ordinary common sense social world, nor even a sketch of what it might be. It is more of a snatch, a glimpse, a doodle for what could be involved; a highly preliminary exercise carried out using as its stalking horse some of the examples we have already discussed. But weak as it is, it suffices for the purposes we have for it. It raises questions about the goodness of fit (relative "inexactness" in Stephen Körner's terms) between our common-sense concepts and the formulations of them in mathematical sociology.

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[8] As Brian Epstein [Epstein 2016] admits, the term is a misnomer. It should be 'social metaphysics'.

[9] For an illustration of this, look no further than Epstein's refutation of 'ontological individualism' by reprising Merton's famous run-on-the-bank thought experiment using the imagined insolvency of Starbucks.

[10] This throws up a separate and but very important corollary to our argument which we do not have space to examine. In Philosophy, conventional metaphysics is the conceptual analysis of how we think about the natural and social worlds. It mostly uses the results of science as its data and reference points. This is justified on the basis of the claim to realism in science (which as we have just seen, in Maddy's eyes rests on shared logical structures). Social ontology is also a 'meta' exercise, this time on Sociology. If there is a divergence of logical structures between Sociology and common sense (which is the thesis we are examining), what does that do to any claim Sociology might make to be structure preserving and hence to the viability of social ontology?

NEIGHBOURHOODS AND PREFERENCES

The two notions which Schelling has to operationalise are 'residential preference' as in 'the character of where I want to live' and 'social or community neighbourhood' as in 'the place where I actually live'. The way he does this is to define neighbourhood as a 'field' of 8 nearest neighbours. We use the notion of 'field' to cover the linear and 2-dimensional spaces he locates neighbourhoods in. Preference is defined as a binary value (+/-) determined by some stipulated tipping point in the (racial) composition of the field.[11]

At first blush it would seem we are on sure ground. After all, neighbourhoods are locales and neighbours live in them. Spatial co-ordinates and counts are paradigm applications of the real number system.[12] Andrew and Lisa live next door and they are our neighbours. Robert and Alison live over the road and they are our neighbours too. But what about Simon and Julie who live a kilometre away? Are they also our neighbours? True there are just two houses between us but calling them neighbours in the same sense Andrew and Alison are doesn't quite work. The reason, of course, has to do with the cluster nature of our concept of neighbour. Sometimes it is used to mean just those who live 'next-door' (to some reasonable numerical value of "next") and sometimes it is used to mean some numerically vague or indeterminate 'living in the general vicinity'. Our village and the next village are indeed neighbouring villages but we don't think of the people living in Warslow as neighbours in anything like the sense Robert and Alison (or even Simon and Julie) might be.

But it gets even trickier. Draw a line around the village. Is the delimited area "our neighbourhood"? Well, for some purposes it is (after all we share a 'neighbourhood shop', a village school and church and other 'neighbourhood services') but for some it isn't. One of the things that plays into this distinction is the expectations we might have of people who live in the neighbourhood. Being 'neighbourly' does not mean greeting everyone we meet on the street and passing the time of day with them. It does not mean feeling obliged to help everyone with clearing snow from the pavement outside their house. For some we do and others we don't. These expectations are not ordered by proximity but are bound up with the multitude of other ties we might have with them. For example, we share friends, they are on the school committee, they collect leaves and petals from our garden for the Well Dressing. Sociologists have bundled all

[11] Most critiques of Schelling focus on the fact the segregation is about race. That is true but not our interest.

[12] Perhaps we ought to add here that the KF-structuring of the basic categories of space and time has been strenuously challenged from Bergson onwards. See [Bergson 2001]

these ways of talking about the people who live near us into the concept of "role". The role "neighbour" is a plastic one. We recognise it, use it, understand it without needing an abstract, formalised descriptive rule for it. Of course, sometimes that usage has an approximate KF in structure as when we talk about the people "over the road" and "next door" So does the designation "neighbouring village". But not all the connotations captured by the sociological role of 'neighbour' are fixed, individuated and denumerable in the same way. And this matters because how we think about the general character of our neighbourhood is a manifold. It depends on what we talking about and to whom we are talking. And that has implications for what might be said about our preferences for living there.

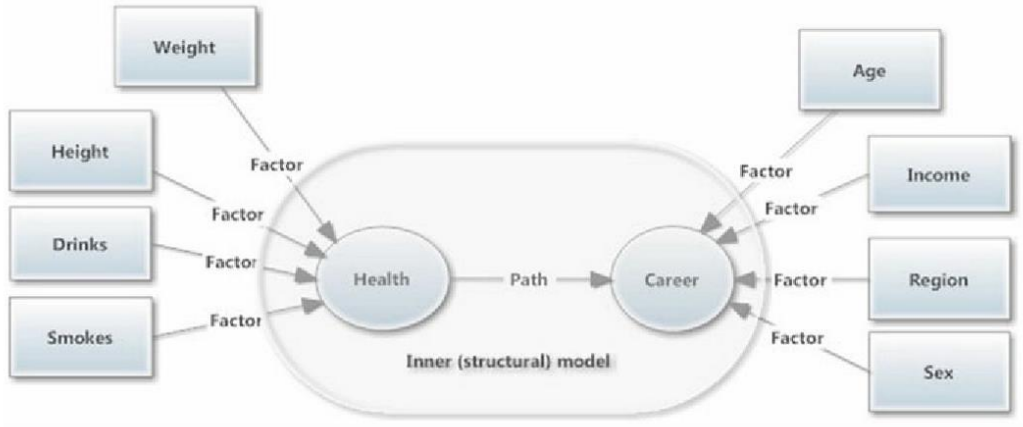
Preference also seems on safe ground. After all, preferences are orderings. You can list your favourite beers and which supermarket in the nearby town you'd rather use. You can give advice on the easiest way to find your house and what determines the choice of routes. But that word 'choice' hides the issues with treating a preference as a fixed, distinct, denumerable 'thing'. You might say the easiest route is to go one way but if traffic is thick, then go another. When traffic is thick, what was the second or third choice is *now* your first preference. Route choosing preferences are circumstance driven. Then think about choosing wine in a restaurant. Someone's preferences might well be fixed: white with fish and only New Zealand. But others might well be open. What do they *feel* like drinking? What do their dinner companions *like*? What are they eating? Choosing now becomes a matter of balancing, weighing, interpreting over many different considerations. To be sure a choice is made, but to say that choice is the *preference* is to stretch that term beyond its limits. Sometimes preferences are KF in form and sometimes they are not just more nuanced but almost the inverse of KF-structures. They are fluid, unstable and have no abstractly discriminable boundaries.

If we want to use the concepts of neighbourhood and preference in a sociological account such as Schelling's, we have at least three choices. We could try to describe how the actions or behaviours we are interested in are shaped by the (perceived) conditions and circumstances in which those who undertake them find themselves. Central to these circumstances is the classic "What the Devil do they think they are up to?" question posed by Geertz. And then the question of 'mattering' we raised earlier becomes important. Alternatively, we could provide arguments for why among the array of analysed usages which make up the cluster our concept conveys, just the KF-structures (or a single KF-structure) are *the relevant ones for this investigation of this phenomenon*. Or third, and this is Schelling's choice, we could decide by fiat to use the term in a way which naturally fits a KF-structure and proceed from there. Schelling's hand was forced by

the need to make the mathematics he wanted to use work. Neither first nor the second option would sustain the kind of analysis he wanted to provide simply because his analysis required arithmetical manipulations.

Healthy Living and Professional Success

Like Schelling, the operationalising Westland undertakes is most easily seen in his visualisations, the path diagrams. In them we can find precisely what is taken to define living healthily and predispose (or, at any rate, mark) having a successful professional career.[13]



The trouble is both these target notions are socially and sociologically complex. For example, how is professional career success made socially and sociologically recognisable? It involves personal attainment, to be sure, signified by promotions and other career markers like salary progression, increased responsibility and authority all wrapped up in increased organisational power. But it also includes peer and even public esteem represented by reputation and acknowledgement of successes in spheres relevant to the domains an individual works within. In many cases, peer and public respect does not track the most obvious markers of 'professional progress'. We had an example before us not so long ago. Those who work in what is called "the front line" of the Covid pandemic were rightly praised for their professionalism, dedication and self-sacrifice. They were celebrated by the media, politicians and the public. But very few of them had the obvious trappings of professional success. They had no insignia (offices, cars, secretaries) and

[13] We are not going to make anything of the difference between these two sets of relationships nor why some indicators have been placed where they are (e.g., why are age and sex not related to health?). Neither will we discuss the fact that the explanation depends upon finding high degrees of association between the indicator variables. The example is an illustrative one and uses standard diagnostic categories from the Health Sciences and Sociology.

were not well paid. The same holds for professionals such as teachers, priests, administrators, authors, musicians and many, many more. In these domains personal and public recognition of success is not marked by easily measurable properties like salary and offices but by expertise, the value others place on the work they do and, most importantly perhaps, by the value they themselves place on it. For these professionals, success is as much intrinsic as extrinsic. Intrinsic (social) characteristics are not easily subjected to standardised measurement. Of course, we don't want to get too starry eyed here. University professors, probation officers, journalists, doctors, nurses and others no doubt do appreciate extrinsic 'reward'. But they are just as sensitive to what their work means to them (well, maybe not always journalists!).

To be fair to Westland, we should not claim he ignores all this. His example is an explication not an actual research report so we cannot say precisely how he would characterise professional success. The indicators he lists are standard demographic variables and all he can produce from them is a description of how whatever he takes success to be varies on them. What we can say, though, is that if he wants to select a sample population on which to measure those demographic variables, the basis on which he chooses who is and who is not successful will face the challenges we outline.

You can run almost the same arguments regarding living healthily. Of course, we understand relative healthiness tends to be associated with various behaviours (non-smoking, moderate alcohol intake, weight control) and height and weight are standard demographic variables. But living a healthy life is as much a personal state of mind as it is the possession of some standard attributes. It has to do with a general sense of personal wellbeing, positive attitude and freedom from anxiety (to pick a few of the many features of our lives we might value). Once again, choosing those who are living healthily cannot be fixed simply by extrinsic, instrumented measures.

GENEROSITY AND THE STATE

Much of what we had to say about Ragin's analysis in *Reading Sociologically* bore upon the topics we are now dealing with. His whole analysis turns on an ordering of countries according to the generosity of their welfare provision. That order falls within a range from 0 to 1 with Sweden, Norway, Denmark being the most generous and USA, Australia and Canada the least. This ranking is the result of comparing the countries with regard to the extent of means testing and the independence of benefits from labour market factors such as earnings, years of work and so on. Labour market considerations and available financial resources are held to be used as throttles on

the level of benefits to which a claimant might be entitled. They are mechanisms for rationing allocations. Countries which have less stringent rationing are more generous than those where the rationing is stricter. Call this the 'open handed conception of generosity'. Now compare it to what we might call the 'Widow's Mite' conception.

> 41 And Jesus sat over against the treasury and beheld how the people cast money into the treasury: and many that were rich cast in much.
>
> 42 And there came a certain poor widow, and she threw in two mites, which make a farthing.
>
> 43 And he called unto him his disciples, and saith unto them, Verily I say unto you, that this poor widow hath cast more in, than all they which have cast into the treasury:
>
> 44 For all they did cast in of their abundance; but she of her want did cast in all that she had, even all her living.

Here, generosity reflects not the characteristics of the receiver but those of the giver. In many ways, this carries more of the commonsense interpretation of generosity than the rationing conception which Ragin's concept carries. This is because the attribution of generosity comes with moral overtones.[14] The widow is deemed to be virtuous because of the proportion of her own wealth given not because of how she determined who should and who should not receive it. No-one would say the widow is less generous because she gives less than others. In fact, the biblical lesson is that she is more generous because she has less to give. One way Ragin might have captured this sense of generosity could be to use the proportion of GDP or of Government spending allocated to welfare provision. The point we are making is that Ragin appears not to have thought what he should mean by generosity, how it should be represented and how whatever representation he uses catches the moral dimension our cultural sense of generosity has. Instead, he taken an off-the-shelf set of 'measures', recast them by means of his ordering mechanism and used them in his analysis.

Section 4. Conclusion

We do not want to overegg the claims made on the basis of the above limited and undoubtedly threadbare analysis. All we would say is that Maddy's analysis of the metaphysics of Physics and Arithmetic might lead us to be diffident about operationalising analytical protocols for sociological concepts around straightforward lifts of arithmetic methods from the natural sciences. There appear

[14] Barbara Kiviat [Kiviat 2023] has recently pointed to the moral implications of using different "narratives" as the basis for constituting social actors as "cases" so they can be subjected to algorithmic-driven processes.

to be cases when simply applying standard arithmetic to counts of some phenomena's occurrence seems to fail to secure a successful closure of the abstraction gap between a piece of sociological analysis and the character of the social experience which it renders. At that point, the social and sociological worlds appear not to align, let alone map.

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